

A new mathematical model for the viscosity of vegetable oils based on freely sliding molecules

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Submitted: 26 August 2018; Accepted: 10 December 2018; Published online: 9 May 2019

SUMMARY: Viscosity is one of the most important parameters for vegetable oil. A new mathematical viscosity model was developed based on freely sliding molecules with Lennard-Jones' potential and linear density-temperature relation. According to the functions derived from the new model, viscosity gradually decreases with temperature at atmospheric pressure. Viscosity increases with density, however, due to the molecular momentum transfer and statistical effect of average molecular potential. After the temperature dependent function is fitted to the 417 experimental data points collected from references and distributed among the ranges: 278.15–453.15 K and 2.6–192.6 mPa.s, it was found that the calculated data agreed well with experimental data with $R^2 \geq 0.961$ for 13 oil samples. The density dependent function was also satisfactorily fitted to the 143 viscosity data points versus density from 5 oil samples with $R^2 \geq 0.917$.

KEYWORDS: Density; Model; Temperature; Vegetable oil; Viscosity

RESUMEN: *Nuevo modelo matemático para la viscosidad de aceites vegetales basado en moléculas de deslizamiento libre.* La viscosidad es uno de los parámetros más importantes para los aceites vegetales. Se desarrolló un nuevo modelo matemático de viscosidad basado en moléculas de deslizamiento libre con potencial de Lennard-Jones y relación densidad-temperatura lineal. Mediante las funciones derivadas del nuevo modelo matemático, la viscosidad disminuye suavemente con la temperatura a presión atmosférica, pero aumenta con la densidad, debido a la transferencia del momento molecular y al efecto estadístico del potencial molecular promediado. La función dependiente de la temperatura se ajusta a los 417 puntos de datos experimentales recopilados de las referencias y distribuidos entre los rangos: 278.15–453.15 K y 2.6–192.6 mPa.s, y se encuentra que los datos calculados concuerdan bien con los datos experimentales con $R^2 \geq 0.961$ para 13 muestras de aceite. La función dependiente de la densidad también se ajusta satisfactoriamente a los 143 puntos de datos de viscosidad frente a la densidad de 5 muestras de aceite con $R^2 \geq 0.917$.

PALABRAS CLAVE: Aceite vegetal; Densidad; Modelo; Temperatura; Viscosidad

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Citation/Cómo citar este artículo: Stanciu I. 2019. A new mathematical model for the viscosity of vegetable oils based on freely sliding molecules. *Grasas Aceites* 70 (3), e318. <https://doi.org/10.3989/gya.0824182>

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1. INTRODUCTION

Vegetable oils are not only consumed as food ingredients in daily life, but also utilized as essential materials in the food industry (Fasina and Colley, 2008; Rodenbush *et al.*, 1999) and in environmentally-friendly products that satisfy the requirements for sustainable development (Belgharza *et al.*, 2014; Mawatari *et al.*, 2013; McNutt and He, 2016). Vegetable oils are usually present as viscous liquids under common application circumstances. Their viscosity property is important to relevant processes like food preparation or processing in homes, restaurants, or food manufacturing (Diamante and Lan, 2014). Viscosity is physically influenced by a set of parameters. Among them, temperature, density and pressure are the most fundamental in the scientific and applied aspects. Therefore, the viscosity of vegetable oil should be expressed as a function of the three parameters,

$$\eta = f(T, \rho, p) \quad (1)$$

here, η , T , ρ , p denote viscosity, temperature, density, and pressure, respectively. At constant pressure, such as at atmospheric pressure, equation (1) can be simplified as,

$$\eta = f(T, \rho) \quad (2)$$

Just as other physical systems, there is an equation of state (the so-called pressure-volume (density)-temperature relationship) for vegetable oil. When the equation of state of the oil at atmospheric pressure is encountered, equation (2) can be further simplified as a univariate function of either temperature or density, which appears as the one-element model to correlate to vegetable oil viscosity.

In contrast to a method which requires time, money and the use of instruments, a correlation to the viscosity value by means of a model is fast and cheap. Therefore, the determination of a mathematical formula of the univariate function, in other words, modeling the viscosity of vegetable oil, has engineering and economic values.

In the history of the research on vegetable oils, there have been considerable attempts to model viscosity with respect to temperature. At atmospheric pressure, the Arrhenius type model equations (3–5) and the Vogel–Fulcher–Tamman (VFT) (Fulcher, 1925; Stanciu, 2011; Tammann and Hesse, 1926; Vogel, 1921) model equation (6) with three parameters, are successful examples. The original and improved forms of these models are listed below.

Arrhenius model:

$$\eta = A \exp(E_A / RT) \quad (3)$$

Improved Arrhenius model:

$$\ln \eta = A + B / T + C / T^2 \quad (4)$$

$$\ln \eta = A + B / T + CT \quad (5)$$

Vogel–Fulcher–Tamman model:

$$\eta = A \exp\left(\frac{T_0}{T - T_1}\right) \quad (6)$$

where, A , B , C , E_A , T_0 , and T_1 are coefficients, R is the universal gas constant.

In addition, the power law model (Fasina *et al.*, 2006; Fasina and Colley, 2008) is occasionally used, which takes this form:

$$\eta = K(T - T_{ref})^n \quad (7)$$

here, K and n are constants, T_{ref} is a reference temperature. Other expanded exponential forms of functions were reported as (Stanciu, 2012; Simion *et al.*, 2014; Valantina *et al.*, 2016):

$$\ln \eta = A + BT + CT^2 \quad (8)$$

$$\eta = \eta_0 + A \exp(-T / B) \quad (9)$$

$$\ln \eta = A + B / T + C \ln T \quad (10)$$

Regarding density, a relation between viscosity and density is very useful in the vegetable oil industry. A viscosity measurement is more complex and time consuming, requiring qualified laboratory technicians. According to the authors' references, few articles report investigations on the relation between viscosity and density for vegetable oils. In 1999, based on an empirical relation between velocity of sound and viscosity in a liquid, Rodenbush *et al.* (Rodenbush *et al.*, 1999) derived the following equation:

$$\frac{U^{1/3}}{\rho} = A + \frac{B}{\eta^{1/2}} \quad (11)$$

where, A and B are coefficients to be fitted, U is the velocity of sound and η is the viscosity in centipoise. They found an absolute average deviation to this formula of 14.6% for 117 experimental

data points. Esteban *et al.*, (2012) used this formula to fit kinematic viscosity (η/ρ), resulting in a smaller absolute average deviation of 1.9% for six vegetable oils. Accordingly, the following formula was used by Esteban *et al.*, (2012):

$$\rho = D + \frac{E}{\eta^{1/2}} \quad (12)$$

where $1/\eta^{1/2}$ is an independent variable.

Up to now, no other viscosity models with respect to density for vegetable oil have been found by the authors of this paper.

The aforementioned models have demonstrated the progress in viscosity modeling for vegetable oils. In spite of the considerable progress made, there is still a gap in the knowledge about the molecular mechanism that governs the variations in vegetable oil viscosity with both temperature and density. In the science and engineering of vegetable oil, it is not only the viscosity model's accuracy and quality that need improving by means of an innovative approach, but knowledge about viscosity also requires deepening into the level of molecular motion and interaction. These are the key problems in the area of viscosity modeling for vegetable oils.

This article is structured as follows: first, a new mathematical model for the viscosity of vegetable oil is developed based on freely sliding molecules with Lennard-Jones' potential and linear density-temperature relation:

$$\phi(r) = 4q[(\sigma/r)^{12} - (\sigma/r)^6] \quad (13)$$

where q and σ define the energy and length scales of the molecule, respectively. The first term represents the repulsive and the second term the attractive part of the intermolecular interaction.

$$T = A\rho^4 - B\rho^2 \quad (14)$$

where A and B are constants which can be fitted separately to the fluid and solid numerical data.

Then, the viscosity variations over either temperature or density are given according to the model. Finally, the quality of the mathematical model is assessed with the literature data measured from more than a dozen vegetable oil samples.

2. MODELS AND CALCULATION METHOD

2.1. New mathematical model for the viscosity of vegetable oils

To build an ideal model for the viscosity of liquid vegetable oil, theoretical analysis is usually required. It is well known that liquid molecules are quite close together. However, the intermolecular forces are not

strong enough to keep neighboring molecules in a fixed position and molecules are free to move past or slide over one another very easily.

In a system like this, there is a relationship between average molecular separation and density. If the volume occupied by each molecule is assumed as the sphere of radius r , and the liquid is closely packed with the spheres, then the average intermolecular separation ($2r$) is able to be estimated by the following equation (Yang and Li, 2015; Yang and Zhu, 2016):

$$\rho = \frac{Nm}{V} = \frac{m}{\frac{4\pi r^3}{3}} \quad (15)$$

where, m denotes molecular mass, N is the total number of molecules contained in volume V .

There is also a relationship between molecular separation and intermolecular potential energy in the freely sliding molecular system. This relationship can be presented by the potential model. A famous potential model is the Lennard-Jones model (1924) (Lennard-Jones, 1924), which takes the following form for a $3\beta-3\alpha$ pattern:

$$\phi = \phi_\sigma \left[\left(\frac{\sigma}{d} \right)^{3\beta} - \left(\frac{\sigma}{d} \right)^{3\alpha} \right] \quad (16)$$

where intermolecular separation is denoted by d , ϕ_σ and σ are the Lennard-Jones' constants.

For the system closely packed with freely sliding spheres, the density can be adopted as the independent variable instead of the intermolecular separation through equation (15) (Yang and Li, 2015; Yang and Zhu, 2016). Then the Lennard-Jones potential at the average intermolecular separation, which is an estimation of the average molecular potential of the system, is written as the summation of repulsive and attractive terms:

$$\phi = \phi_\sigma \left[\left(\frac{\rho}{\rho_\sigma} \right)^\beta - \left(\frac{\rho}{\rho_\sigma} \right)^\alpha \right] \quad (17)$$

here, $\rho_\sigma = 6m/(\pi\sigma^3)$, stands for a characteristic density at which the potential equals zero.

Using the density dependent Lennard-Jones' potential, symmetric molecular interaction approximation (Yang and Li, 2015; Yang and Zhu, 2016) and Maxwell-Boltzmann distribution function for the freely sliding molecules, liquid viscosity can be derived as:

$$\eta = K\sqrt{T}\rho^{\frac{2}{3}} \exp\left[\frac{1}{T}(A\rho^\alpha - B\rho^\beta)\right] \quad (18)$$

where K , A , B are coefficients which are independent of density and temperature. This equation presents the mathematical model for viscosity in terms of both temperature and density without a restriction on pressure. Physically, the equation (18) involves two molecular mechanisms that govern the variations in viscosity with temperature and density (Yang and Zhu, 2016). The first is the momentum transfer due to the transportation of the freely sliding molecules, which is achieved through the term in the form of root square of temperature multiplying 2/3rd power of density. The second is the statistical effect of molecular potential realized in terms of the exponent of the ratio of average intermolecular potential to average kinetic energy. With respect to liquid vegetable oil at atmospheric pressure, the equation of state is found as a negatively linear relation between density and temperature (Esteban *et al.*, 2012), which can be expressed as:

$$\rho = \xi(T_0 - T) \quad (19)$$

or

$$T = \frac{1}{\xi}(\rho_0 - \rho) \quad (20)$$

here, ξ and T_0 or $\rho_0 = \xi T_0$ are coefficients.

Based on this equation of state, equation (19) is able to be simplified as a univariate function. When the independent variable is temperature, substituting equation (20) into equation (21) yields:

$$\eta = K\sqrt{T}(T_0 - T)^{\frac{2}{3}} \exp\left[\frac{A(T_0 - T)^\alpha - B(T_0 - T)^\beta}{T}\right] \quad (21)$$

where, K , A , B , α , β , are adjusting coefficients, T the temperature in K, η the viscosity in mPa.s. This equation presents a new mathematical model for vegetable oil viscosity as the function of temperature at atmospheric pressure. When the independent variable is density, substituting equation (20) into equation (18) obtains:

$$\eta = K\rho^{\frac{2}{3}}\sqrt{\rho_0 - \rho} \exp\left[\frac{A\rho^\alpha - B\rho^\beta}{\rho_0 - \rho}\right] \quad (22)$$

here, ρ stands for the density in g/cm³, η the viscosity in mPa.s. This equation gives a new model for vegetable oil viscosity versus density at atmospheric pressure. Equations (21) and (22) are firstly derived upon the linear relation between density and temperature and an innovative approach that can deepen into the level of molecular motion and interaction. Therefore, it may be said that a considerable theoretical advancement has been made in modeling the viscosity of vegetable oil. Molecular mechanisms

that govern the variations in vegetable oil viscosity with both the temperature and density have been found, and thus the aforementioned knowledge gap has been filled. For these reasons, the viscosity model's quality and calculating accuracy are expected to be further improved with the two equations.

2.2. Calculation method

After a model has been constructed, a comparison against experimental data is usually required to assess the quality of the model. During the assessment, comprehensive statistical methods may be employed, for example, the curve fit with the least square method, the R squared determination, computer software utilization, and so on. In the present study, a non-linear robust fitting was used and carried out with the software named TableCurve 2D v5.01.

In statistics, the R^2 of a fit for a model to measured data is a number that indicates the explanatory power for the model to explain the data. It is thus an ideal parameter to quantitatively demonstrate the performance of the author's new model. The higher the degree of the amount of R^2 approaching 1 may suggest more internal laws of vegetable oil involved in the model.

3. RESULTS

3.1. Temperature dependence

Calculated viscosity versus temperature. In order to visualize the variation in viscosity with temperature by the author's model, equation (19) was numerically calculated when $\ln K = -4.3$, $A = -0.1$, $B = -8 \times 10^{-5}$, $\beta = 3$, $T_0 = 500$, and $\alpha = 1, 1.4, 1.6$.

Consequently, three viscosity curves were obtained and plotted in figure 1 during the temperature increase from 250 to 500 K. They show that

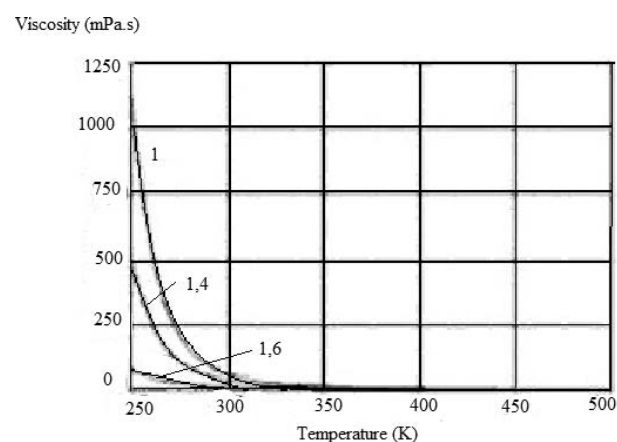


FIGURE 1. Dynamic viscosity versus absolute temperature according to equation (21) as $\alpha = 1, 1.4, 1.6$

the viscosity quickly decreases with temperature in a low temperature region, in contrast to the small decreasing rate in the high temperature region. This trend qualitatively agrees with the commonly accepted knowledge about the viscosity of vegetable oil. The lower α value leads to higher viscosity at a given temperature, indicating a high sensitivity for the molecular interaction parameter to alter the viscosity (Gupta *et al.*, 2007; Nouredini *et al.*, 1992).

In order to compare the new model with existing models, the 61 viscosity data points of soybean oil and 58 data points of corn oil were used again. The comparing samples included the 7 models presented in the introduction section. The fitted results from

all the models are listed in Table 3, which shows that the new model gives the fits of $R^2=0.988$ and 0.997 for the two oils. These two numbers not only mean the comprehensively best performance among the models in question, but also the considerable improvement in the model's accuracy and quality.

3.2. Density dependence

Calculated curves of density dependent viscosity.

As mentioned in the introduction section, the viscosity as function of density is very useful in the vegetable oil industry. The equation (20) from the author's model can give a theoretical prediction to the variation viscosity with density. When the coefficients of equation (20) took the amounts:

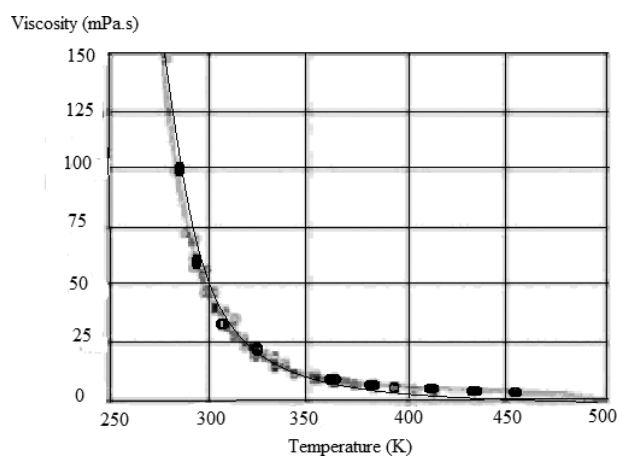


FIGURE 2. Viscosity of soybean oil versus temperature at atmospheric pressure. Squares denote gathered experimental data, while the curve is calculated by equation.21 with the coefficients in Table 2.

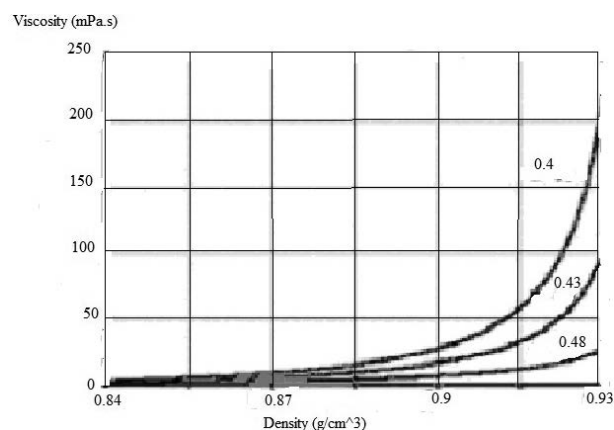


FIGURE 3. Curves of viscosity versus density according to equation 22 as $\alpha=0.4, 0.43, 0.48$.

TABLE 1. Experimental viscosity dynamic data of vegetable oils with respect to temperature and references

Oil	Points	Temperature range (K)	Viscosity range (mPa.s)	References
Soybean	61	278.15-453.15	3.3100-147.8000	6 references ^a
Corn	58	278.15-453.15	3.3000-138.3000	5 references ^b
Sunflower	56	278.15-453.15	3.5200-155.0000	6 references ^a
Grapeseed	47	278.15-453.15	2.6000-165.2000	6 references ^a
Almond	30	278.15-453.15	3.6200-173.0000	3 references ^c
Rapeseed	24	283.15-413.15	3.7500-110.0000	3 references ^c
Olive	21	278.15-453.15	3.4400-182.9000	3 references ^c
Palm	20	298-413.15	3.6500-82.6000	3 references ^c
Canola	20	278.15- 453.15	4.2900-166.2000	3 references ^c
Peanut	20	278.15-453.15	3.2600-192.6000	3 references ^c
Safflower	20	278.15-453.15	3.4400-125.0000	3 references ^c
Walnut	20	278.15-453.15	3.4600-116.6000	3 references ^c
Sesame	20	278.15-453.15	3.4300-155.2000	3 references ^c
Overall	417	278.15-453.15	2.6000-192.6000	6 references ^a

^a Fasina *et al.*, 2008; Diamante *et al.*, 2014; Fasina *et al.*, 2006; Simion *et al.*, 2014; Esteban *et al.*, 2012; Nouredini *et al.*, 1992

^b Fasina *et al.*, 2008; Diamante *et al.*, 2014; Fasina *et al.*, 2006; Simion *et al.*, 2014; Esteban *et al.*, 2012

^c Fasina *et al.*, 2008; Diamante *et al.*, 2014; Fasina *et al.*, 2006

TABLE 2. Fitting results of equation (21) to the viscosity data versus temperature from relevant references

Oil	Ln(K)	A	B	T0	α	β	R ²
Soybean	-4.3000	-0.1360	-1.02×10^{-4}	502	1.4500	3	0.9880
Corn	-5.1000	-0.0655	-5.75×10^{-5}	542	1.2700	3	0.9970
Sunflower	-4.4200	-0.1220	-8.67×10^{-5}	512	1.4200	3	0.9900
Grapeseed	-3.6600	-0.3080	-1.58×10^{-4}	512	1.5400	3	0.9610
Almond	-3.9500	-0.0850	-1.43×10^{-4}	469	1.5000	3	0.9920
Rapeseed	-6.2300	0.0673	1.04×10^{-5}	558	1.7800	3	0.9750
Olive	-4.2300	-0.0642	-1.04×10^{-4}	488	1.3900	3	0.9970
Palm	-6.1000	0.4520	-2.09×10^{-5}	541	1.3800	3	0.9880
Canola	-4.5100	-0.1170	-7.23×10^{-5}	540	1.4800	3	0.9990
Peanut	-3.9000	-0.0757	-1.44×10^{-4}	466	1.4700	3	0.9940
Safflower	-4.3100	-0.0494	-9.07×10^{-5}	489	1.4000	3	0.9990
Walnut	-4.5600	-0.0716	-7.22×10^{-5}	513	1.3900	3	0.9990
Sesame	-4.3800	0.0524	-8.71×10^{-5}	497	1.3300	3	0.9990

Viscosity (mPa.s)

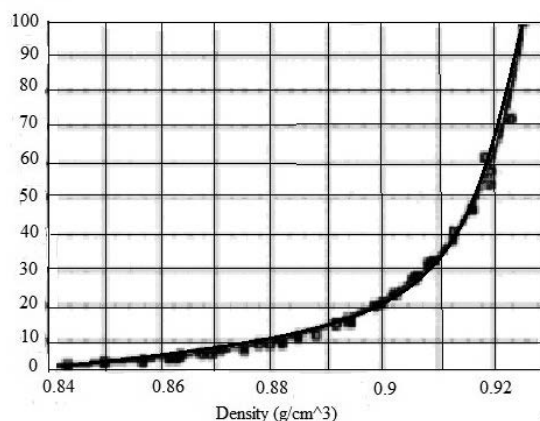


FIGURE 4. Viscosity of soybean oil with respect to density at atmospheric pressure. Squares denote gathered experimental data, while the curve is calculated by equation 22 with the coefficients in Table 5.

$\ln K = -17.3$, $A = 8.45$, $B = 9.50$, $\beta = 3$, $\rho_0 = 0.953$, and $\alpha = 0.40, 0.43, 0.48$, three curves of viscosity versus density were calculated as Figure 3 in the density range from 0.84 to 0.93 g/cm³. According to this figure, it is easily found that viscosity increases with density. This nature has been commonly accepted for vegetable oils. The increasing rate raises in the high density region. At a fixed density, the higher α value corresponds to the lower viscosity value.

Fits to the density dependent model. As above, soybean oil was chosen as the first sample to be fitted. The fitted results are presented in both Figure 4 and Table 5. The results show that the curve by equation (20) meets most of the 34 data points with the R² value of 0.994, suggesting a good fit and high explanatory ability for the author's model to the experimental viscosity data of soybean oil. Similar

TABLE 3. Fitted R² from comparing models to the viscosity dynamic data of soybean and corn oils with respect to temperature absolute

Model or Formula	Soybean	Corn
This work	0.9880	0.9970
$\eta = \text{EXP}(A + B/T)$	0.9770	0.9920
$\eta = \text{EXP}(A + B/T + C/T^2)$	0.9880	0.9960
$\eta = \text{EXP}(A + B/T + C \cdot T)$	0.9860	0.9970
$\eta = \text{EXP}(A + B/(T - C))$	0.9870	0.9960
$\eta = A(T - 273.15)^B$	0.9380	0.9130
$\eta = \text{EXP}(A + B/T + C \cdot \ln(T))$	0.9870	0.9960
$\eta = \eta_0 + A \cdot \text{EXP}(-T/B)$	0.9800	0.9950

good fits were also found with the other four vegetable oils listed in Table 5. The minimum R squared value is 0.917, which shows how the author's new model can successfully describe the experimental viscosity data versus density, and then can include the major internal laws of the vegetable oil.

Comparison to existing density dependent models.

The density dependent models reported for vegetable oil are much fewer than the temperature dependent ones. For the mentioned density dependent models in the introduction section plus a power law model, comparisons were made against the model of the present work using soybean and corn oils.

The results are illustrated in Table 6, which shows that equation (20) of the author's new model considerably improves the accuracy and quality, and comprehensively performs the best among these models.

3.3. Limitation

It has been demonstrated that the new model developed upon the freely sliding molecules with Lennard-Jones' potential and linear

TABLE 4. Experimental viscosity dynamic data versus density and references

Oil	Points	Density (g/cm ³)	Viscosity range (mPa.s)	References
Soybean	34	0.8430-0.9250	3.3700-99.6000	3 references ^a
Corn	34	0.8420-0.9240	3.6600-105.0000	3 references ^a
Sunflower	27	0.8410-0.9250	3.6700-110.0000	2 references ^b
Grapeseed	27	0.8440-0.9260	2.6000-93.2000	2 references ^b
Rapeseed	21	0.8400-0.9210	3.7500-110.0000	2 references ^c
Overall	143	0.8400-0.9260	2.6000-110.0000	3 references ^a

^a Simion *et al.*, 2014; Esteban *et al.*, 2012; Noureddini *et al.*, 1992^b Simion *et al.*, 2014; Esteban *et al.*, 2012^c Esteban *et al.*, 2012; Noureddini *et al.*, 1992

TABLE 5. Fitting results from equation (20) to the viscosity data versus density from relevant references

Oil	Ln(K)	A	B	ρ_0	α	β	r^2
Soybean	-17.3	8.45	9.5000	0.9530	0.4300	3	0.9940
Corn	-17.1	8.30	9.3000	0.9530	0.4310	3	0.9790
Sunflower	-10.1	5.39	5.4200	0.9810	0.4810	3	0.9910
Grapeseed	-10.0	5.11	4.8600	0.9930	0.4800	3	0.9890
Rapeseed	-9.53	6.69	3.2500	1.1600	1.0500	3	0.9170

TABLE 6. Fitted R^2 from comparing mathematical models to the viscosity data of soybean and corn oils with respect to density

Model or Formula	Soybean	Corn
This work	0.9940	0.9790
$\rho = A + B\eta^{-0.5}$	0.9930	0.9780
$\rho = A + B(\eta/\rho)^{-0.5}$	0.9930	0.9790
$\eta = A(\rho - \rho_0)^B$	0.9710	0.9620

density-temperature relation can fit both the temperature and density dependent experimental data of vegetable oils in good agreement and comprehensively performs the best among the models in question. The good fits and out-performances propose an inclusion of the major internal laws of the vegetable oil into the model, and therefore the above-presented molecular mechanism that governs the variations of vegetable oil viscosity has been certified from a point of view. The goodness and out-performance also suggest considerable progress in the improvements to the accuracy and quality of the viscosity model for vegetable oil over broad temperature, density and viscosity ranges. The parameter ranges cover the common industrial working circumstances so as to encompass considerable application potentials of the new model in industry associated with vegetable oils. It may also be stated that the new model suits the usual vegetable oils and their mixture, biodiesel, and other liquid oils. However, it cannot be applied to crystalline oil, or other systems composed of either large molecular polymers or inorganic molecules.

4. DISCUSSION

The authors of this paper have gathered experimental viscosity data versus temperature for the vegetable oils listed in Table 1 from the literature, so as to be able to evaluate the developed mathematical model. According to this table, 417 viscosity data points determined at atmospheric pressure in the temperature and viscosity ranges from 278.15

to 453.15 K, 2.6 to 192.6 mPa.s, are measured with 13 oil samples: soybean, corn, sunflower, grapeseed, almond, rapeseed, olive, palm, canola, peanut, safflower, walnut, sesame oils.

Soybean oil is one of the most widely consumed cooking oils, and a typical representative of vegetable oils. Hence, it becomes an ideal specimen to be used to assess the quality of the developed model. There are 61 viscosity data points gathered from the 7 references listed in Table 1 over a temperature range from 278.15 to 453.15 K and viscosity range from 3.31 to 147.8 mPa.s at atmospheric pressure. They were redrawn in Figure 2, where it can be seen that a smooth curve by equation (19) fitted to the data, centrally penetrates the area of dense data points. The R^2 of the fit is 0.988, a value very close to 1, implying a good fit and high explanatory power of the author's new model to the experimental data for soybean oil.

Analogically good fits were found for the other vegetable oils listed in Table 1. The squared R values are presented in Table 2, which shows that equation (19) can precisely describe the variations in these oil samples with $R^2 \geq 0.961$. This feature suggests a success in modeling viscosity versus temperature for the 13 vegetable oils and an inclusion of the major internal laws of the vegetable oil into the model. Also, the coefficients listed in Table 2 with the equation (19) have significance in correlating the viscosity of the corresponding vegetable oil in engineering applications.

Although the experimental viscosity data for vegetable oils simultaneously tested with density

are relatively fewer than the data versus temperature, the authors of this paper gathered 143 data points for five oils: soybean, corn, sunflower, grape-seed, and rapeseed among the density and viscosity ranges: 0.840–0.926 g/cm³, 2.6–110 mPa.s (as listed in Table 4). These data are suitable to evaluate the developed model, when density is adopted as an independent variable.

5. CONCLUSIONS

A new model for vegetable oil viscosity as a function of either temperature or density was developed based on the freely sliding molecules with Lennard-Jones' potential and the linear density-temperature relation of vegetable oil. According to the new model, viscosity gradually decreases with temperature in contrast to an increase with density, due to the molecular momentum transfer and statistical effect of averaged molecular potential. After the temperature dependent function of the model was assessed with 417 experimental data points collected from references, which were distributed among the temperature and viscosity ranges: 278.15–453.15 K and 2.6–192.6 mPa.s, it was found that the calculated data agreed well with the experimental data with $r^2 \geq 0.961$ for all the 13 oil samples. The density dependent function was also satisfactorily fitted to the 143 viscosity data points versus density from 5 oil samples with $r^2 \geq 0.917$. This goodness of fit means the success and high explanatory power of the new model, as well as the inclusion of the major internal laws of vegetable oil into the model. In comparison with the other models in question, the new model outperforms, and upgrades accuracy and quality. In addition, the theoretical backgrounds for molecular sliding and interaction have academic superiority and entail a considerable advancement in modeling vegetable oil viscosity. Since the new model has given successful formulae to estimate viscosity values of the mentioned vegetable oils, it has application potentials in the industry associated with vegetable oils. It can save time and money and provide a convenient approach for technologists to determine the viscosity values in the design, analysis, and production processes related to these vegetable oils.

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